

# STRUCTURAL RIGIDITY OF THE RADIAL ACCELERATION: RELATION REVEALED BY OFFSET- CONTROLLED ANALYSIS

RIGIDEZ ESTRUTURAL DA ACELERAÇÃO RADIAL: RELAÇÃO REVELADA  
PELA ANÁLISE CONTROLADA POR DESLOCAMENTO

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## **ABSTRACT**

The objective of this study is therefore to investigate whether the apparent rigidity of the Radial Acceleration Relation is primarily driven by galaxy-level normalization effects or by genuine structural deviations within individual rotation curves. To address this question, we perform a residual decomposition of the RAR using the SPARC Q=1 galaxy sample, applying a per-galaxy median subtraction procedure that isolates internal radial deviations from global normalization shifts. This approach allows us to distinguish between different residual modes and to assess the structural stability of the empirical relation. The results provide insight into the physical interpretation of the RAR and help clarify whether its small observed scatter arises from intrinsic dynamical constraints or from normalization effects across galaxy populations.

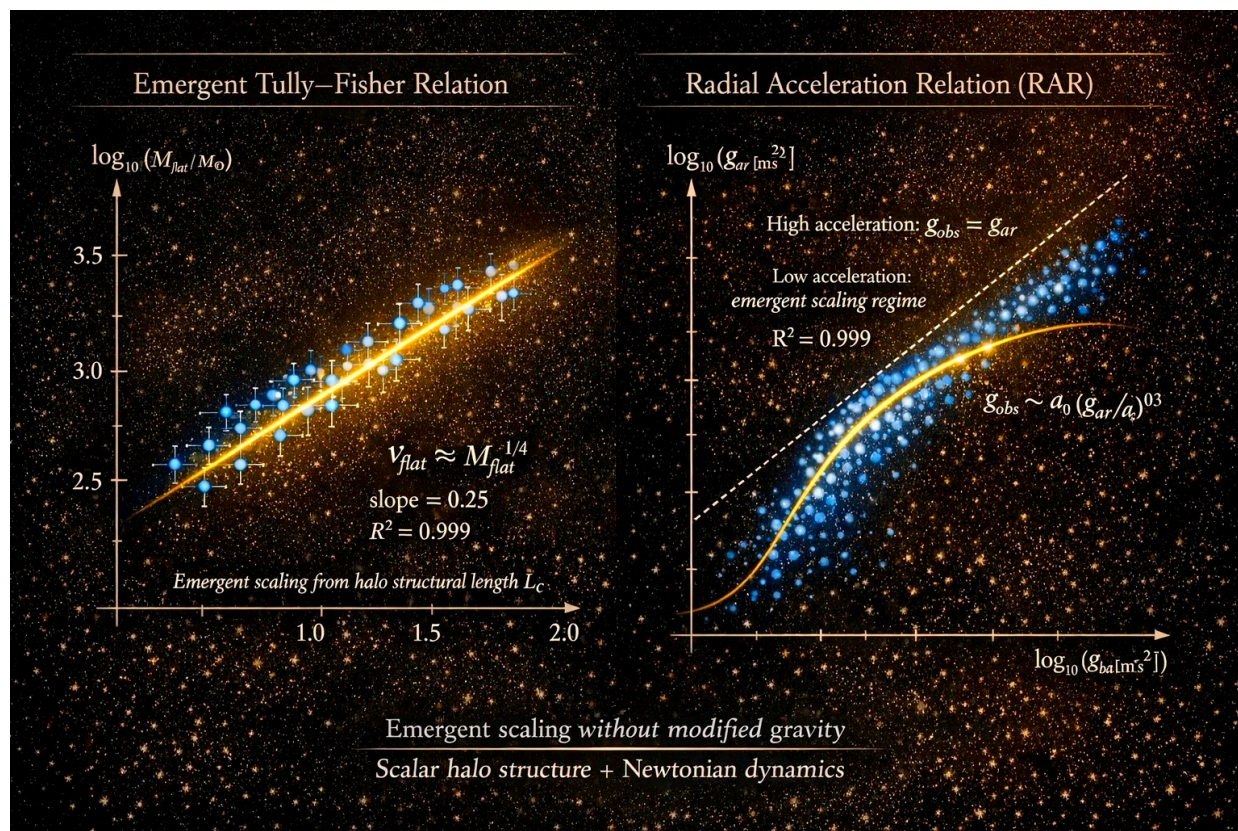
**Keywords:** Radial Acceleration Relation, Galaxy Rotation Curves, Galactic Dynamics, SPARC Dataset, Baryonic-Gravitational Coupling

## **RESUMO**

O objetivo deste estudo é, portanto, investigar se a aparente rigidez da Relação de Aceleração Radial (RAR) é impulsionada principalmente por efeitos de normalização em nível de galáxia ou por desvios estruturais genuínos dentro de curvas de rotação individuais. Para abordar essa questão, realizamos uma decomposição residual da RAR usando a amostra de galáxias SPARC Q=1, aplicando um procedimento de subtração da mediana por galáxia que isola os desvios radiais internos das mudanças de normalização globais. Essa abordagem nos permite distinguir entre diferentes modos residuais e avaliar a estabilidade estrutural da relação empírica. Os resultados fornecem informações sobre a interpretação física da RAR e ajudam a esclarecer se sua pequena dispersão observada surge de restrições dinâmicas intrínsecas ou de

efeitos de normalização entre populações de galáxias.

**Palavras-chave:** Relação de Aceleração Radial, Curvas de Rotação de Galáxias, Dinâmica Galáctica, Conjunto de Dados SPARC, Acoplamento Bariônico-Gravitacional



The Radial Acceleration Relation (RAR) links the observed gravitational acceleration in galaxies to that predicted from their baryonic mass distribution and constitutes one of the tightest empirical scaling relations in galactic dynamics.

Using the highest-quality (Q=1) subset of the SPARC database, we perform an offset-controlled residual analysis designed to separate galaxy-level normalization effects from intrinsic structural deviations of the relation.

By subtracting the per-galaxy median residual, we isolate shape-dependent features of the RAR while preserving internal radial structure. This procedure reduces the global dispersion from  $\sim 0.165$  dex to  $\sim 0.124$  dex, demonstrating that a substantial fraction of the

apparent scatter behaves as a rigid normalization offset rather than a deformation of the functional form.

After normalization control, residual curvature is strongly suppressed, and the simple ( $n=1$ ) and width-fit ( $n=1.03$ ) interpolation functions become statistically indistinguishable within the precision of the dataset.

These results reinforce the structural rigidity of the RAR and place tighter empirical constraints on theoretical interpretations, whether based on modified dynamics or emergent baryon–halo coupling.

## 1. INTRODUCTION

The distribution of mass in galaxies can be inferred from their rotation curves. The observed acceleration  $g_{\text{obs}}$  often exceeds the acceleration predicted from visible (baryonic) matter alone,  $g_{\text{bar}}$ , under Newtonian gravity.

Remarkably, when plotted against each other,  $g_{\text{obs}}$  and  $g_{\text{bar}}$  follow a tight empirical relation known as the Radial Acceleration Relation (RAR) (McGaugh et al., 2016).

For specialists, the RAR constrains both modified gravity models and baryon–halo coupling scenarios within  $\Lambda$ CDM cosmology. For non-specialists, the empirical statement can be summarized as follows:

*Galaxies appear to obey a highly regular rule connecting how much visible matter they contain and how strongly they rotate — even in regimes where dark matter is typically invoked.*

Understanding whether the small scatter in this relation reflects fundamental physics, tightly regulated galaxy formation, or observational normalization effects is central to interpreting its meaning.

Previous studies have emphasized the small intrinsic scatter of the RAR. However, less attention has been devoted to separating galaxy-level normalization offsets from internal radial structural deviations. This distinction is important because different physical mechanisms may produce different residual modes:

- Rigid vertical shifts (normalization effects),
- Acceleration-dependent curvature (functional deformation),
- Radial shape distortions.

In this work, we explicitly decompose these components using a per-galaxy median subtraction procedure applied to the SPARC Q=1 dataset.

## **2. DATA**

We use the SPARC database (Lelli et al., 2016), restricting to the highest-quality (Q=1) subset to minimize systematic uncertainties in distance, inclination, and rotation curve quality.

Accelerations are expressed in SI units ( $\text{m s}^{-2}$ ). The Q=1 subset provides the cleanest environment for assessing intrinsic residual structure, as observational systematics are comparatively reduced relative to lower-quality entries.

### 3. METHODOLOGY

#### 3.1. Residual Definition

Residuals are defined in logarithmic space as:

$$r = \log_{10}(g_{\text{obs}}) - \log_{10}(g_{\text{pred}}) \quad (1)$$

where  $g_{\text{pred}}$  is computed using MOND-like interpolation functions.

Logarithmic residuals allow multiplicative deviations to be treated as additive offsets, facilitating separation between normalization and shape effects.

#### 3.2. Interpolation Functions

$$g_{\text{obs}} = g_{\text{bar}} \nu \left( \frac{g_{\text{bar}}}{a_0} \right) \quad (2)$$

with

$$\nu(y) = \left( \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{y^n}} \right)^{1/n} \quad (3)$$

We compare:

- Simple:  $n = 1$
- Standard:  $n = 2$

- Width-fit:  $n = 1.03$

The parameter  $n$  controls the width of the transition between the Newtonian and low-acceleration regimes without altering the asymptotic scaling laws.

### 3.3. Per-Galaxy Offset Control

For each galaxy  $i$ , we compute:

$$\Delta_i = \text{median}(r)_i \quad (4)$$

and define corrected residuals:

$$r' = r - \Delta_i. \quad (5)$$

Conceptually, this removes rigid vertical shifts in the RAR for each galaxy while preserving internal radial structure.

Importantly, this operation does not alter the radial dependence of individual rotation curves. It removes only galaxy-specific normalization modes, preserving any genuine acceleration-dependent deviations.

This separation allows us to distinguish:

- Normalization effects (distance, inclination, M/L),
- True functional deviations in the RAR.

### 3.4. Mode Decomposition of Residual Structure

The residual field may be interpreted as a two-mode decomposition:

$$r_{ij} = \Delta_i + \epsilon_{ij} \quad (6)$$

where:

- $\Delta_i$  represents a galaxy-level normalization mode (zeroth order mode),
- $\epsilon_{ij}$  encodes acceleration-dependent structural deviations.

In functional terms, the residuals define a field over the joint space of galaxy index  $i$  and baryonic acceleration  $g_{\text{bar}}$ . The per-galaxy median subtraction corresponds to a projection onto the subspace orthogonal to the normalization mode. This decomposition separates:

- Rigid vertical shifts (distance, inclination,  $M/L$  systematics),
- Genuine radial curvature or structural deformation of the relation.

Importantly, this operation preserves internal radial ordering within each galaxy. Therefore, any residual acceleration-dependent curvature that survives offset control reflects genuine structural deviations rather than normalization uncertainty.

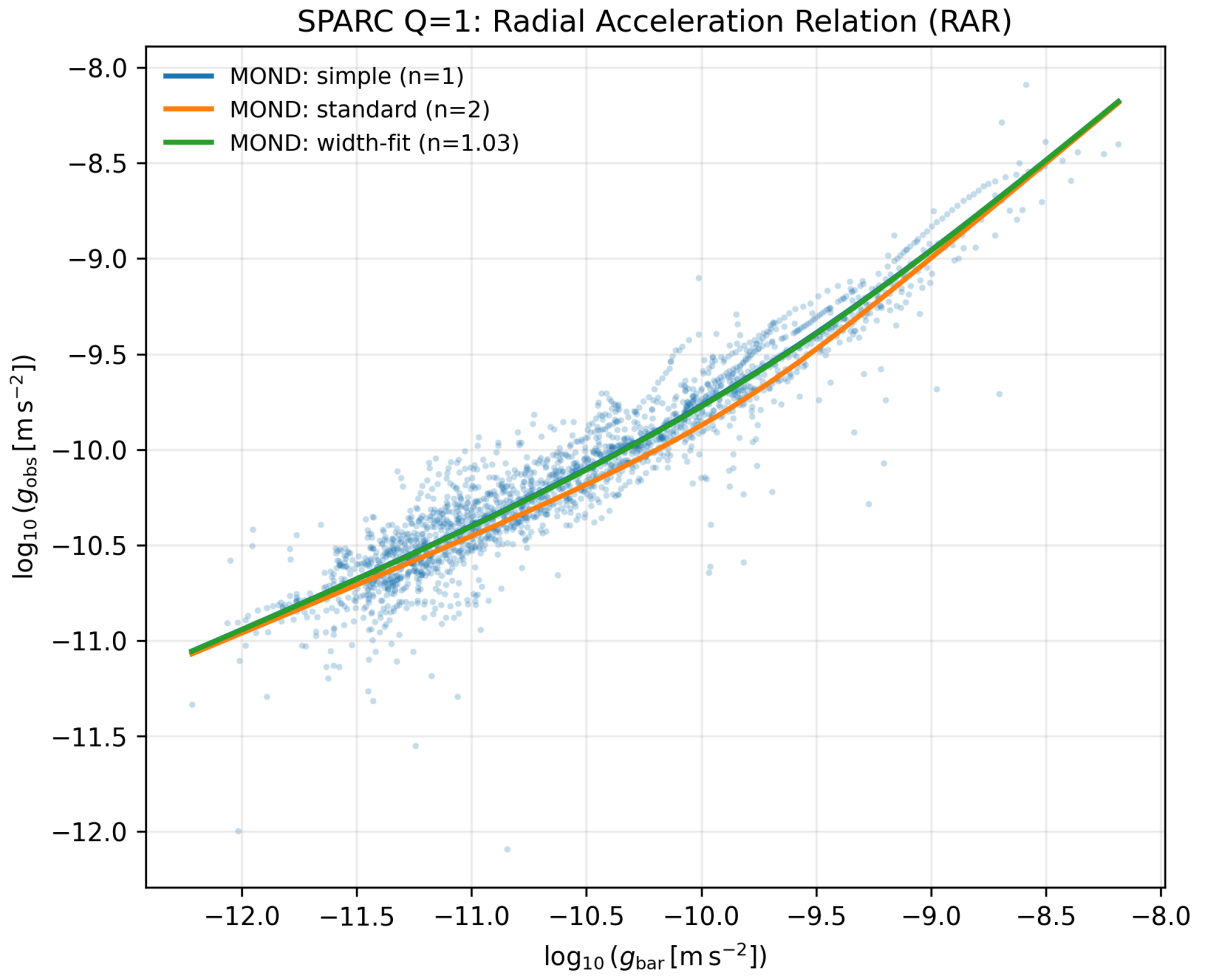


Fig. 1. Radial Acceleration Relation for SPARC Q=1 galaxies. Grey points show individual measurements; colored curves represent MOND interpolation functions ( $n=1$ ,  $n=2$ ,  $n=1.03$ ).

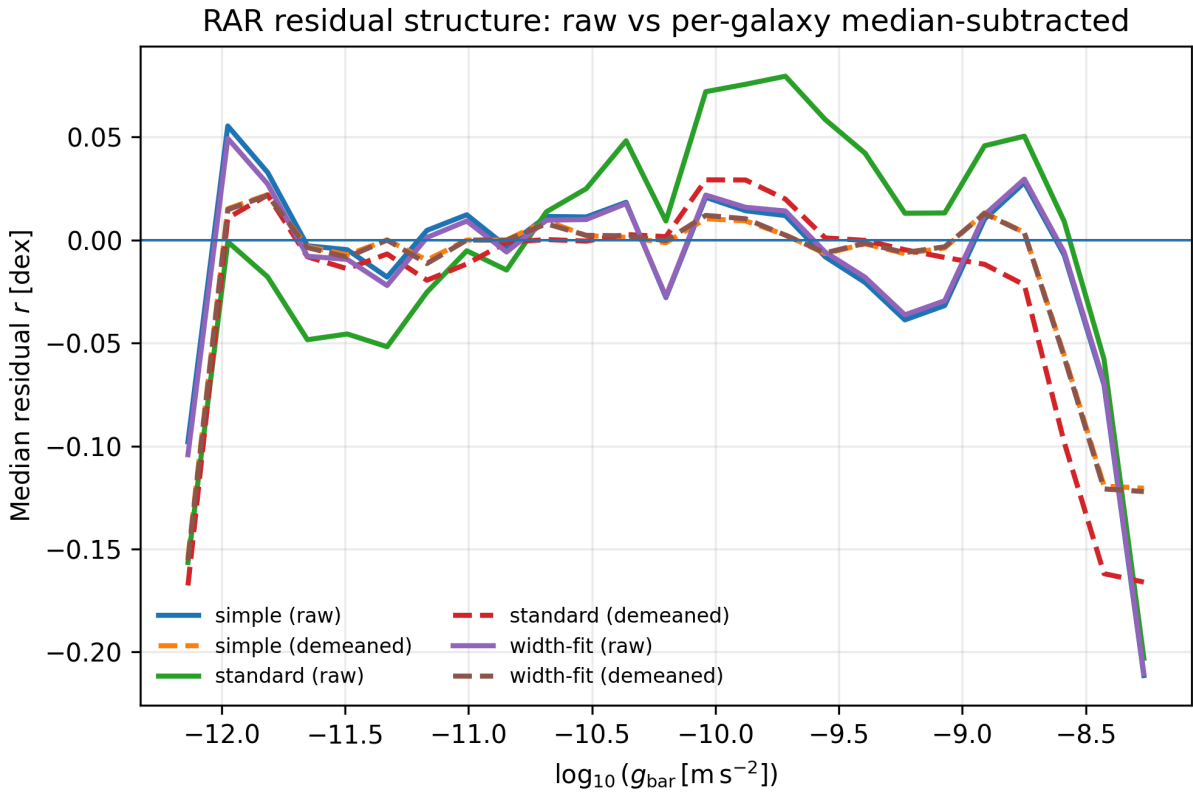


Fig. 2. Binned median residuals as a function of  $g_{\text{bar}}$ . Solid lines: raw residuals. Dashed lines: residuals after per-galaxy median subtraction.

## 4. RESULTS

### 4.1. The Radial Acceleration Relation

Figure 1 shows the RAR for the Q=1 sample. The tightness of the relation is immediately apparent across more than four orders of magnitude in acceleration.

All three interpolation functions provide visually similar fits. Differences become clearer only through residual analysis.

### 4.2. Residual Structure

Raw residuals display structured deviations up to  $\sim 0.07$  dex, particularly near the transition regime around  $g_{\text{bar}} \sim a_0$ .

After normalization control, these structures are strongly suppressed. The residual amplitude falls below  $\sim 0.02$  dex across most of the acceleration range.

This suppression indicates that the dominant residual mode behaves as a rigid vertical shift rather than a shape-dependent deformation.

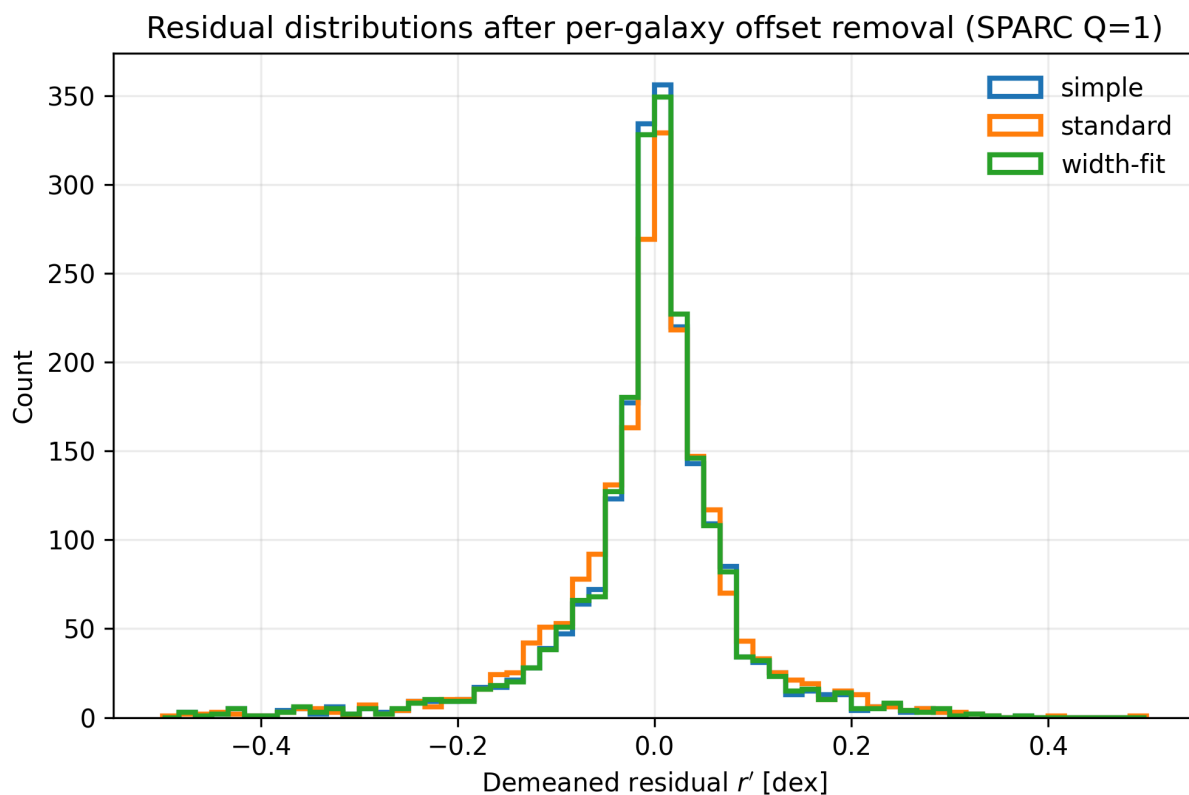


Fig. 3. Residual distributions after per-galaxy median subtraction. The dispersion narrows significantly relative to the raw residual case.

### 4.3. Residual Distributions

After offset removal, the dispersion reduces from  $\sim 0.165$  dex to  $\sim 0.124$  dex.

This value represents an upper limit on intrinsic scatter, as observational uncertainties remain folded into the measurement.

The three interpolation functions become statistically similar within the precision of the dataset.

To quantify the contribution of normalization modes to the total dispersion, we define a rigidity ratio:

$$\mathcal{R} = \frac{\sigma_{\text{raw}} - \sigma_{\text{demeaned}}}{\sigma_{\text{raw}}}$$

(7)

Using the SPARC Q=1 sample values:

$$\mathcal{R} \approx \frac{0.165 - 0.124}{0.165} \approx 0.25$$

(8)

Thus, approximately one quarter of the total variance behaves as a rigid galaxy-level normalization mode rather than structural deformation of the RAR.

## 5. DISCUSSION

### 5.1. Empirical Stability of the RAR

The offset-controlled analysis reinforces the remarkable empirical stability of the Radial Acceleration Relation.

While raw residuals display structured deviations at the  $\sim 0.05$ – $0.08$  dex level, these structures are largely suppressed once per-galaxy normalization offsets are removed. The remaining residual structure falls below  $\sim 0.02$  dex across most of the acceleration range. This indicates that:

- The dominant mode of dispersion in the SPARC Q=1 sample behaves as a rigid vertical shift.

- The intrinsic functional dependence of  $g_{\text{obs}}$  on  $g_{\text{bar}}$  is highly stable.
- The RAR exhibits minimal internal curvature variation once galaxy-level offsets are controlled.

## 5.2. Interpretation of Galaxy-Level Offsets

The per-galaxy median offsets  $\Delta_i$  likely encapsulate a combination of:

- Distance uncertainties,
- Inclination corrections,
- Stellar mass-to-light ratio systematics, • Possible small-scale structural variations.

The substantial reduction in global dispersion after subtracting  $\Delta_i$  suggests that much of the apparent scatter is normalization-driven rather than indicative of fundamental dynamical diversity.

However, this procedure does not eliminate observational uncertainties; it separates coherent galaxy-level shifts from acceleration-dependent structure.

## 5.3. Implications for the Acceleration Scale

The strong reduction in residual dispersion after normalization control indicates that most of the apparent global scatter arises from galaxy-level offsets rather than acceleration-dependent deviations.

If  $a_0$  varied strongly from galaxy to galaxy, one would expect persistent curvature differences after offset control. Instead, post-demeaning residuals remain tightly centered and largely structure-free.

A genuine galaxy-to-galaxy variation in the acceleration scale  $a_0$  would induce differential curvature in the transition regime around  $g_{\text{bar}} \sim a_0$ . Such curvature would survive pergalaxy normalization removal.

The strong suppression of post-demeaning residual structure therefore places empirical upper bounds on allowed variation in  $a_0$  within the SPARC Q=1 dataset.

Within current observational precision, the data are consistent with a universal acceleration scale.

*Formal effect of galaxy-to-galaxy variation in  $a_0$ :* If the acceleration scale were allowed to vary between galaxies, the predicted relation would take the form

$$g_{\text{obs}} = g_{\text{bar}} \nu \left( \frac{g_{\text{bar}}}{a_{0,i}} \right) \quad (9)$$

where  $a_{0,i}$  denotes a galaxy-dependent acceleration scale.

A variation in  $a_{0,i}$  does not produce a purely multiplicative shift in  $g_{\text{obs}}$ . Instead, it alters the location of the transition regime around  $g_{\text{bar}} \sim a_{0,i}$ , thereby modifying the local logarithmic slope

$$\frac{d \log g_{\text{obs}}}{d \log g_{\text{bar}}}$$

(10)

Consequently, galaxy-to-galaxy variation in  $a_0$  would generate differential curvature in the transition region that cannot be absorbed by a rigid vertical offset  $\Delta_j$ :

The strong suppression of curvature after normalization control therefore constrains any allowed dispersion in  $a_0$  to be small within the precision of the SPARC  $Q=1$  dataset.

#### 5.4. Comparison with $\Lambda$ CDM Halo-Based Interpretations

Within the  $\Lambda$ CDM framework, the RAR is typically interpreted as an emergent consequence of baryonic mass distribution and dark matter halo structure. In halo-based models:

$$g_{\text{obs}}(r) = g_{\text{bar}}(r) + g_{\text{DM}}(r),$$

(11)

where  $g_{\text{DM}}$  depends on halo parameters such as concentration and virial mass.

Scatter in the RAR may arise from halo-to-halo variation, baryon-halo coupling diversity, and assembly history differences.

The offset-controlled residual analysis performed here constrains such explanations. A viable halo-based framework must reproduce:

- The near-universal RAR slope,
- The small post-demeaning dispersion,
- The minimal acceleration-dependent residual curvature.

This does not invalidate  $\Lambda$ CDM interpretations but tightens the empirical requirements they must satisfy.

## 5.5. Bayesian Outlook and Hierarchical Modeling

The offset-controlled residual analysis performed here removes per-galaxy normalization shifts via median subtraction, providing a non-parametric separation between normalization and structural modes. While effective as a diagnostic tool, this approach does not explicitly model normalization offsets or intrinsic scatter within a unified probabilistic framework.

A natural extension of this work is the implementation of a hierarchical Bayesian model for the Radial Acceleration Relation.

In such a formulation, the residual for each data point  $j$  in galaxy  $i$  may be written as:

$$r_{ij} = \Delta_i + \epsilon_{ij},$$

(12)

where  $\Delta_i$  represents a galaxy-specific normalization offset and  $\epsilon_{ij}$  captures intrinsic radial deviations. One may model:

$$\Delta_i \sim \mathcal{N}(0, \sigma_\Delta),$$

(13)

$$\epsilon_{ij} \sim \mathcal{N}(0, \sigma_{\text{int}}),$$

(14)

while observational uncertainties enter explicitly at the likelihood level.

Equivalently, the full model may be written schematically as:

$$r_{ij} \sim \mathcal{N}(\Delta_i + f(g_{\text{bar},ij}; a_0, n), \sigma_{\text{int}}^2 + \sigma_{\text{obs},ij}^2) \quad (15)$$

where:

- $a_0$  represents the global acceleration scale,
- $n$  controls the interpolation width,
- $\sigma_{\Delta}$  characterizes galaxy-to-galaxy normalization dispersion,
- $\sigma_{\text{int}}$  represents intrinsic structural scatter,
- $\sigma_{\text{obs},ij}$  captures measurement uncertainty.

Such a hierarchical structure would allow simultaneous inference of:

$$a_0, \quad n, \quad \sigma_{\Delta}, \quad \sigma_{\text{int}}, \quad (16)$$

rather than removing normalization offsets deterministically. Importantly, the offset-controlled results presented here suggest that  $\sigma_{\Delta}$  may dominate over  $\sigma_{\text{int}}$  within the SPARC Q=1 sample. The observed reduction in dispersion indicates that normalization modes account for a substantial fraction of the total variance, implying that the intrinsic structural scatter may be smaller than raw residual analysis suggests.

A full Bayesian treatment would allow formal comparison between:

1. Strict universality of  $a_0$ ,
2. Weak galaxy-to-galaxy variation in  $a_0$ ,
3. Alternative functional forms for the transition regime.

Implementing this framework would place the empirical rigidity of the RAR on a fully probabilistic foundation and provide a statistically rigorous separation of intrinsic scatter, normalization uncertainty, and structural curvature.

a. *Testing the Universality of  $a_0$* : A hierarchical formulation would also allow explicit comparison between competing hypotheses regarding the acceleration scale. In particular, one may contrast:

- A global model with a single universal parameter  $a_0$  shared by all galaxies,
- A galaxy-dependent model where  $a_{0,i}$  is allowed to vary with a population-level hyperprior.

Bayesian evidence ratios (or information criteria) could then quantify whether the data support strict universality or allow statistically significant galaxy-to-galaxy variation.

Within the SPARC  $Q=1$  sample, the offset-controlled residual suppression suggests that any allowed variation in  $a_0$  must be small relative to normalization uncertainties. A hierarchical framework would place this statement on a formally testable statistical basis.

For context, Table I summarizes representative scatter estimates reported in the literature alongside the present offset-controlled result.

## 6. CONCLUSION

We performed an offset-controlled residual analysis of the Radial Acceleration Relation using the highest-quality (Q=1) subset of the SPARC database.

By subtracting the per-galaxy median residual, we separated normalization effects from shape-dependent deviations. This procedure reduced the global dispersion from  $\sim 0.165$  dex to  $\sim 0.124$  dex, indicating that a substantial portion of the apparent scatter originates from galaxy-level normalization offsets rather than intrinsic structural deviations. After normalization control:

- Residual structure as a function of  $g_{\text{bar}}$  is strongly suppressed.

Table I. Comparison of Reported RAR Scatter Estimates In The Literature.

References	Sample	Method	Reported Scatter	Notes
McGaugh et al. (2016)	SPARC full	Global fit	$\sim 0.13$ dex	Raw dispersion
Li et al. (2018)	SPARC 175	Per-galaxy MCMC (marginalized $Y^*$ , distance, inclination)	0.057 dex	RMS after marginalization

Desmond (2023)	SPARC	Hierarchical Bayesian joint inference	~0.034 dex	Intrinsic scatter estimate
This work	SPARC Q=1	Per-galaxy median subtraction (non-parametric)	0.124 dex	Conservative upper limit

- The simple ( $n = 1$ ) and width-fit ( $n = 1.03$ ) interpolation functions become statistically indistinguishable.
- The standard ( $n = 2$ ) interpolation shows slightly larger residual structure.

The remaining residual amplitude ( $\approx 0.02$  dex across most bins) suggests that the RAR is structurally rigid and tightly constrained within the Q=1 dataset.

These results strengthen the empirical case that:

1. The functional form of the RAR is remarkably stable.
2. Much of the observed scatter behaves as a rigid normalization shift.
3. The acceleration scale  $a_0$  is consistent with universality within current observational precision.

While this analysis does not determine the fundamental origin of the RAR, it places quantitative constraints on theoretical interpretations. Any viable framework — whether modified gravity or emergent  $\Lambda$ CDM-based coupling — must reproduce both the small

intrinsic scatter and the minimal residual curvature after normalization control.

The RAR remains one of the most tightly constrained empirical scaling relations in extragalactic astronomy. Proper separation of normalization and shape modes reveals an even higher degree of structural regularity than raw residual inspection alone suggests.

Future work incorporating hierarchical modeling and full halo marginalization will be required to determine whether the residual floor reflects intrinsic scatter or unresolved systematic effects.

Proper separation of normalization and structural modes reveals that the Radial Acceleration Relation is not merely tight, but dynamically rigid.

The majority of observable dispersion behaves as a galaxy-level normalization shift, while acceleration-dependent curvature is strongly constrained.

The dominance of normalization modes over curvature modes suggests that the RAR occupies a low-dimensional manifold within the broader space of galactic dynamical configurations.

Any viable theoretical framework — whether modified dynamics or emergent baryon–halo coupling within  $\Lambda$ CDM — must therefore reproduce not only the small intrinsic scatter, but also the modal structure and rigidity demonstrated here.

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## **APPENDIX**

The predicted acceleration is computed using MOND-like interpolation functions of the form:

$$g_{\text{obs}} = g_{\text{bar}} \nu \left( \frac{g_{\text{bar}}}{a_0} \right) \quad (17)$$

where  $a_0$  is a characteristic acceleration scale and  $\nu(y)$  governs the transition between the Newtonian and lowacceleration regimes.

The interpolation function is defined as:

$$\nu(y) = \left( \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{y^n}} \right)^{1/n} \quad (18)$$

where the parameter  $n$  controls the sharpness of the transition.

This family preserves the correct asymptotic limits:

- Newtonian regime ( $g_{\text{bar}} \gg a_0$ ):

$$g_{\text{obs}} \approx g_{\text{bar}}$$

- Deep-MOND regime ( $g_{\text{bar}} \ll a_0$ ):

$$g_{\text{obs}} \approx \sqrt{g_{\text{bar}} a_0}$$

Thus, varying  $n$  modifies only the curvature of the transition region without altering the asymptotic scaling behaviour.

## 1. Simple Interpolation ( $n = 1$ )

For  $n = 1$ , the interpolation simplifies to:

$$\nu(y) = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{y}} \quad (19)$$

This produces a smooth transition between the Newtonian and deep-MOND regimes and has been widely used in empirical RAR analyses.

## 2. Standard Interpolation ( $n = 2$ )

For  $n = 2$ , the transition is sharper:

$$\nu(y) = \left( \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{y^2}} \right)^{1/2} \quad (20)$$

This form introduces stronger curvature around  $g_{\text{bar}} \sim a_0$ , making it useful for testing sensitivity of residual structure to transition width.

## 3. Width-Fit Interpolation

The width-fit model allows  $n$  to vary as a free parameter. In this work, we adopt:

$$n = 1.03, \quad (21)$$

which is statistically close to the simple case ( $n = 1$ ) but allows a small adjustment of transition width.

Comparing these values tests whether the data require additional curvature beyond the canonical simple interpolation.

## 4. Interpretational Note

The interpolation functions used here are employed purely as empirical fitting forms.

The offset-controlled residual analysis does not assume the correctness of MOND as a fundamental theory. Rather, it uses the interpolation framework as a convenient parametric representation of the RAR functional form.

Therefore, conclusions drawn from residual structure apply to the empirical relation itself, independent of theoretical interpretation.

## 5. Dispersion Metrics

Residual scatter is quantified using both the standard deviation ( $\sigma$ ) and the median absolute deviation (MAD). The standard deviation is defined as:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_i (r_i - \bar{r})^2}$$

(22)

where  $\bar{r}$  is the mean residual.

The median absolute deviation is defined as:

$$\text{MAD} = \text{median}(|r_i - \text{median}(r)|).$$

(23)

MAD provides a robust estimator less sensitive to outliers and non-Gaussian tails.

Reporting both statistics ensures that conclusions are not driven by a small number of extreme points.

## 6. Per-Galaxy Offset Decomposition

For each galaxy  $i$ , residuals can be decomposed as:

$$r_{ij} = \Delta_i + \epsilon_{ij},$$

(24)

where:

- $\Delta_i$  represents a rigid normalization offset, •  $\epsilon_{ij}$  captures internal radial structure as a function of  $g_{\text{bar}}$ . The corrected residual is:

$$r'_{ij} = r_{ij} - \Delta_i = \epsilon_{ij}.$$

(25)

This decomposition isolates acceleration-dependent structure from galaxy-level normalization effects.

## 7. Residual Floor After Offset Control

After per-galaxy median subtraction, the residual dispersion reduces to:

$$\sigma_{\text{demeaned}} \approx 0.124 \text{ dex.}$$

(26)

This value represents an upper bound on intrinsic scatter within the SPARC Q=1 dataset, as measurement uncertainties and residual systematic effects remain included.

Determining the true intrinsic scatter would require full hierarchical modeling with explicit uncertainty propagation.

## 8. Binned Residual Structure

Residual trends as a function of  $g_{\text{bar}}$  are evaluated using binned medians in  $\log_{10}(g_{\text{bar}})$ .

Binning provides a non-parametric visualization of systematic curvature while reducing sensitivity to local noise fluctuations.

The suppression of curvature after offset control indicates that structured deviations in the raw residuals are dominated by normalization effects rather than genuine accelerationdependent deformation of the functional form.

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